

## Comparison of Clark and Geographical Instantaneous Unit Hydrograph Models for Arid and Semi Arid Regions

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**Abstract:** Rainfall runoff models play a significant role in water resources engineering, planning, management and development. Groundwork of these models and identification of their parameters however, is a challenging task for arid and semi arid catchments having scanty data. There are several types of rainfall runoff models. This paper compares the results obtained from two hydrologic techniques namely Clark and Geographical Instantaneous Unit Hydrograph. The model parameters were identified by optimization based on downhill simplex technique. The models were applied to catchments with short interval floods in a semi-arid region of Pakistan. Computer based program has been developed for each model. Satellite imageries of the area and rainfall data were collected from National Engineering Services of Pakistan (NESPAK) and Punjab Irrigation and Power Department. Physical parameters of the catchments like land slope, delineation, associated drainage areas and stream lengths have been deducted from SPOT satellite images of the catchment area using Arc GIS and ERDAS. Discharge data was estimated by measuring parameters of channel from the field. The models provided acceptable results for peak discharge and also for the time to peak of the hydrograph. The Clark's technique resulted to be better as compared to the Geographical Instantaneous Unit Hydrograph technique.

**Key words:** Clark • Geographical • Hydrograph • Catchment • Instantaneous • Parameters

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### INTRODUCTION

Water resources engineering, planning, management and development is important for sustainability of almost all the countries world wide. However, simulation of hydrologic responses of catchments is highly complex especially for arid and semiarid catchments. Spatial and temporal distributions of rainfall as well as properties of soil and geomorphology affect the direct runoff from a watershed. Arid and semi-arid regions usually have intense rainfall events with high degree of spatial variability coupled with flash floods. Most of the arid and semi arid regions have low rain gauge density and rarely have radar coverage. Scanty data in such situations puts limits on the capacity of rainfall-runoff model to generate the flows representing the real response of the catchment. Calibration and validation of models for such regions is a major problem because of fewer rainfall and

runoff events as compared to watersheds in the wetter ones. Research is needed both in advancing the tools and in applying the existing tools for solution of the real life problems. Practitioners, design engineers and managers are usually interested in work on applying the existing tools for solution of the real life problems. Work done by Ahmed [1], Johnston and Kummur [2], Ghumman *et al.* [3] and Ghumman *et al.* [4] is worth mentioning regarding application of tools for solution of practical problems.

A variety of rainfall runoff models are in use worldwide. Simple models relating runoff to only a few parameters are easy to handle but such models do not provide accurate results. The detailed models on the other hand can provide better accuracy but have more parameters. Identification of these parameters is highly difficult. Large and accurate data is required for identification of model parameters.

Unit hydrograph is one of the commonly used techniques for modeling rainfall-runoff process. A theoretical relationship is developed between geomorphology and hydrology by geomorphologic instantaneous unit hydrograph (GIUH) and the direct runoff reaction of the watershed to unit excess rainfall. Controlling time distribution for channels and terrestrial planes, non periodic relationships between flow velocities and peak discharges, the excess rainfall depth at a specified time, the watershed morphology etc are the factors used in different ways by different researchers to build GIUH, determine model parameters and estimate the direct runoff hydrograph (DRH) from GIUH. Research in this regard by Horton [5], Clark [6], Nash [7], Gupta *et al.* [8], Rosso [9], Chow *et al.* [10], Rodriguez-Iturbe and Valdes [11] and Singh [12] can be considered as pioneer work. Kumar *et al.* [13] has worked on estimation of Clark and Nash model parameters which is a complex task. They assumed peak of geographical instantaneous unit hydrograph and estimated the Clark and Nash model parameters for a catchment area of 1191 square kilometer. The model efficiency was found to be in the range of 38.5 to 94.3% in case of Clark model and 48.3% to 97.9% for Nash Model. High level of uncertainty with geomorphologic descriptors was found. Sarangi *et al.* [14] evaluated Clark model and Semi distributed unit hydrograph (UH) for a catchment area of 26 km<sup>2</sup> in Canada. The error in these models was in the range of 4.6 to 62.1%. They proved that the catchment size play an important role in simulation process. Ahmad *et al.* [15], Ahmad *et al.* [16], Ghumman *et al.* [4] and Ghumman *et al.* [3] has presented work using one of the Clark, Nash or GIUH models.

This research was conducted on simultaneous application of two models for real data to compare the performance of these models on catchment in semi arid region having flash floods and hill torrents.

**Study Area:** Water resources of Pakistan play a dynamic role in its sustainability. Arid and semi-arid zones located in the country face a problem due to storms having high peaks for a very short time. Hence it was useful to apply the models for simulation of runoff hydrograph. The area under study consists of hill ranges and valley floors in Dera Ghazi (DG) Khan district as shown in figure 1. Kaha is a large catchment in the region. A few perennial irrigation schemes called Karezes exist in the area. These originate from aquifers having low flows amounting to 0.02 m<sup>3</sup>/s. Karezes are also charged by flows from hill torrents. Local farmers build temporary diversions to harness flood flows for cultivating crops. Rugged terrain

and sharp bends in natural streams make the geomorphology of the area complex. No detailed mapping is available as the area is scarcely populated and very difficult to access due to non-availability of proper road infrastructure. However presence of satellite imageries of the area and GIS environment processing (ERDAS) has made possible to know the geomorphologic details of this area. The catchment area consists of barren mountains up to 65%. The streams carry short interval flows only during rains.

## MATERIALS AND METHODS

**Clark's Model:** Clark's model can be represented by the following equation, Clark [6]:

$$Q_{i+1} = 2C_o R_{E(i)} + C_i Q_i \quad (1)$$

Here  $i$  is the value of index changing from 1 to N whereas N represents the number of ordinates of the time area diagram as given below in detail,  $R_E$  is uniformly distributed rainfall excess,  $Q_{i+1}$  represents the  $(i+1)^{th}$  ordinate of the Clark's Instantaneous Unit Hydrograph,  $C_o$  and  $C_i$  are known as Muskingham's weighting constants as given below:

$$C_o = 0.5t/(R+0.5t) \quad (2)$$

and

$$C_i = (R-0.5t)/(R+0.5t) \quad (3)$$

In these equations  $t$  is computational time interval and  $R$  is the storage coefficient.

The time-area diagram shows the relationship between time taken by rain water to reach the basin exit and the area of the catchment contributing to generate the direct runoff. The time taken by rain water to reach the basin exit is also known as the time of concentration  $T_c$ . The only experimental variable in time-area diagram is the time of concentration of the respective runoff to the basin exit. It can be given by Kirpich formula as follows [17]:

$$(T_j) = 0.06628 L_j^{0.77} / S_j^{0.385} \quad (4)$$

The whole catchment area is divided into sub-basins.  $(T_j)$  in above equation is concentration time in hours for  $j^{th}$  sub-basin,  $j$  is index changing from 1 to M where M represents the number of identified sub basins.  $L_j$  is length of stream of  $j^{th}$  sub basin in kilometers and  $S_j$  is respective land slope of the sub basin  $j$ .

The time of concentration ( $T_c$ ) and storage coefficient ( $R$ ) are the main parameters of Clark's model which were identified using optimization technique in this paper.

$$R_A = \left( \frac{\bar{A}_\omega}{A_1} \right)^{\frac{1}{\omega-1}} \quad R_B = [N_\omega] \Omega^{-\omega} \quad \text{and} \quad R_L = \left( \frac{\bar{L}_\omega}{L_1} \right)^{\frac{1}{\omega-1}} \quad (7)$$

**Geographical Instantaneous Unit Hydrograph (GIUH):**

Nash's model parameter in form of watershed characteristics can be given as [9]:

$$n = 3.29 \left( \frac{R_B}{R_A} \right)^{0.78} R_L^{0.07} \quad (5)$$

$$k = 0.7 \left( \frac{R_A}{R_B R_L} \right)^{0.48} \left[ \frac{L_\Omega}{V} \right] \quad (6)$$

Here 'n' and 'k' are the Nash model parameters,  $R_A$ ,  $R_B$ ,  $R_L$  are the Horton's ratios [5],  $L_\Omega$  is length of highest order tributary (km) and  $V$  is peak velocity (m/s). The parameter k is in hours. The values of  $R_A$  and  $R_B$  are given below:

Here  $\omega$  represents the order of the stream,  $\Omega$  is order of the basin determined from systematic ordering of streams,  $\bar{A}_\omega$  is the average area contributing to the stream of order  $\omega$ ,  $N_\omega$  represents the number of streams of order  $\omega$ ,  $\bar{L}_\omega$  is average length of streams of order  $\omega$  and  $L_1$  is average length of streams of first order.

The only unknown variable in equation (6) is  $V$  which can be estimated by the equation derived by Zelazinski [18] as:

$$V = \alpha(Q_{max})^\beta \quad (8)$$

Here  $Q_{max}$  is the peak flow for an event of rainfall. The  $\alpha$  and  $\beta$  are the parameters determined from a set of discharge measurements at the outfall of the watershed.

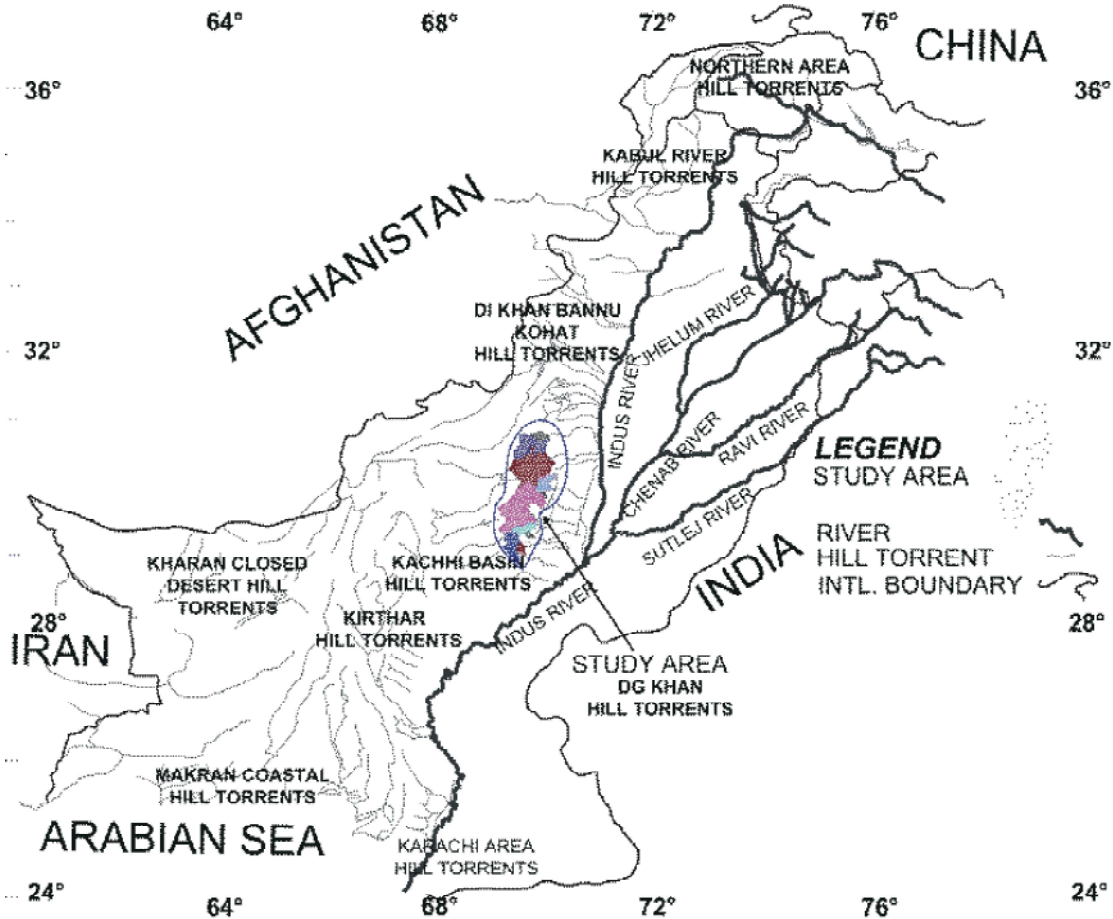


Fig. 1: Map showing location of the study area.

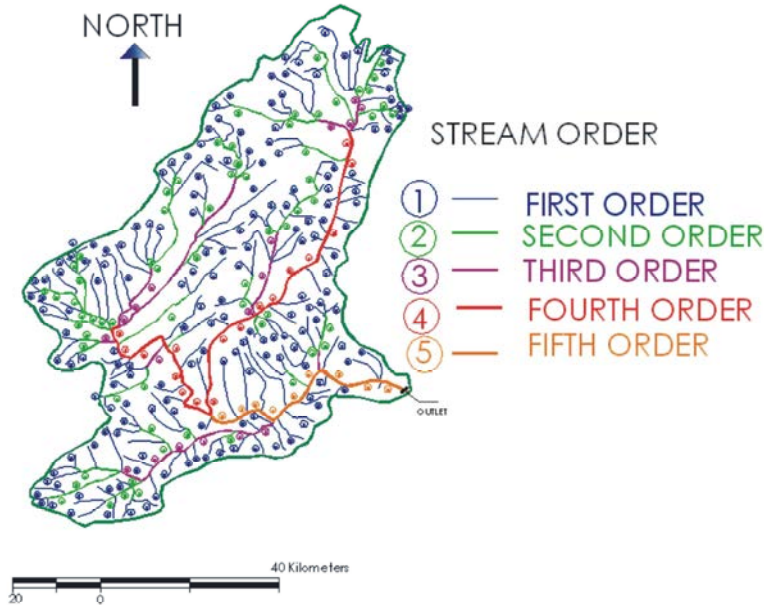


Fig. 2: Configuration of drainage and stream order in study area.

In this paper the values of  $\alpha$  and  $\beta$  were determined by optimization using the data obtained from various flood events.

**Digitization of Catchment Map and Data Processing:**

The physical parameters of the catchments have been estimated from SPOT satellite images of the catchment area. Arc GIS and ERDAS was used to processes the data of the watershed for determining the geomorphologic information like the area of the catchment, the length of highest order stream, total length of streams, longest stream length and stream order of the catchment (as shown in Figure 2).

**Observed Rainfall-runoff Data and Abstractions:**

Irrigation and Power Department is responsible for the management of the hill torrents in Pakistan.

The last fifteen years data regarding rain and runoff was obtained from this department. The losses were subtracted from the total rainfall depth to get the excess rainfall. The percentage runoff technique [19] was used for estimation of excess rainfall. As the study area is un-gauged, so the peak discharge was derived from the survey of cross-sections having the highest flood marks.

**RESULTS AND DISCUSSIONS**

The sample direct runoff hydrographs obtained from the models are presented in Figure 3 to Figure 8. Comparison between observed and computed values of peak flows by Clark and GIUH is given in figure 9 and figure 10. The features of the direct runoff hydrograph considered for comparison are: percent difference between the observed and simulated peak flow ( $\Delta Q_p$ ),

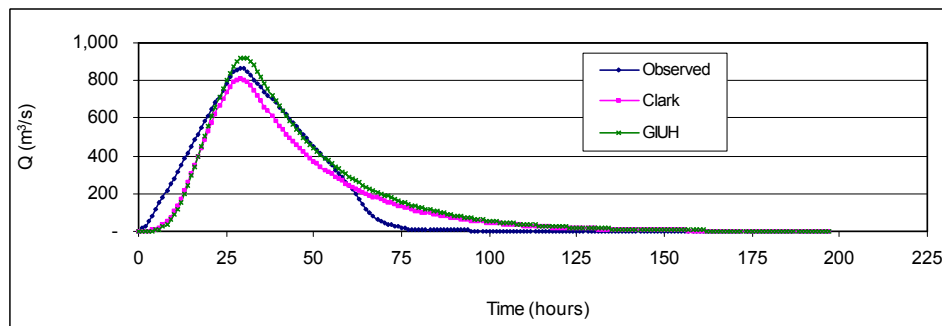


Fig. 3: Comparison between observed and computed direct runoff hydrographs (E1)

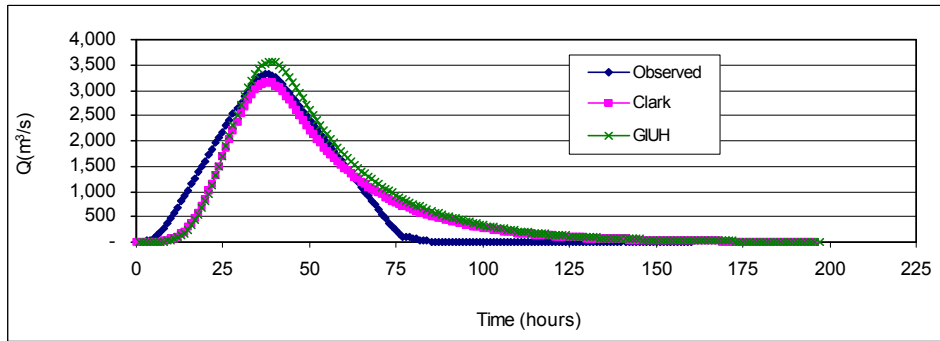


Fig. 4: Comparison between observed and computed direct runoff hydrographs (E2)

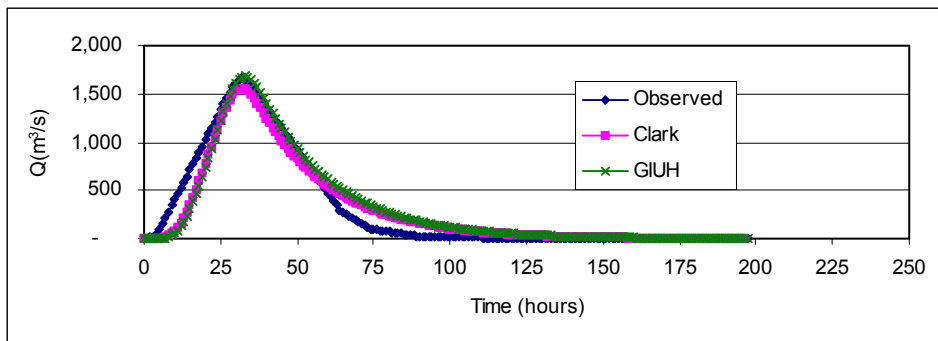


Fig. 5: Comparison between observed and computed direct runoff hydrographs (E3)

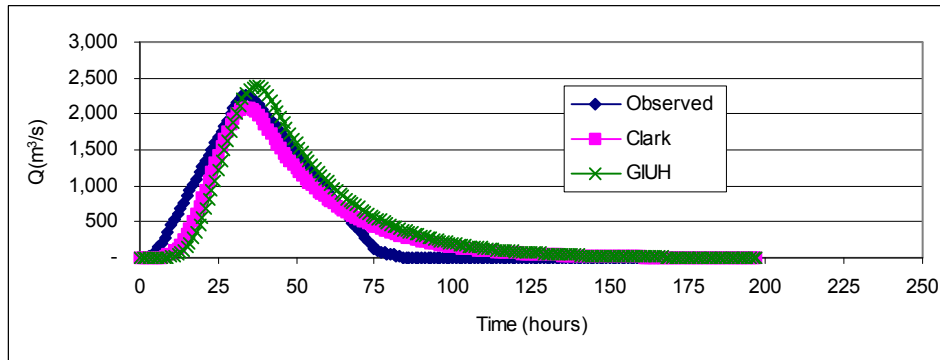


Fig. 6: Comparison between observed and computed direct runoff hydrographs (E10)

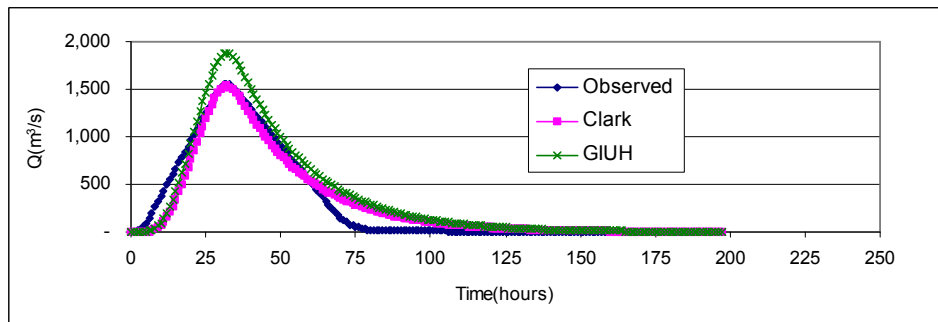


Fig. 7: Comparison between observed and computed direct runoff hydrographs (E11)

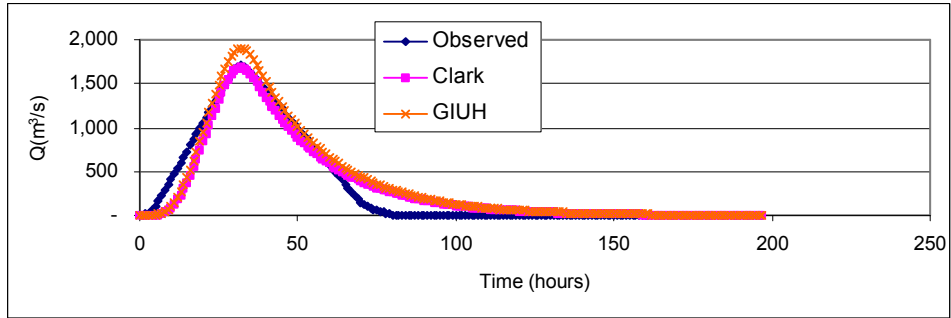


Fig. 8: Comparison between observed and computed direct runoff hydrographs (E12)

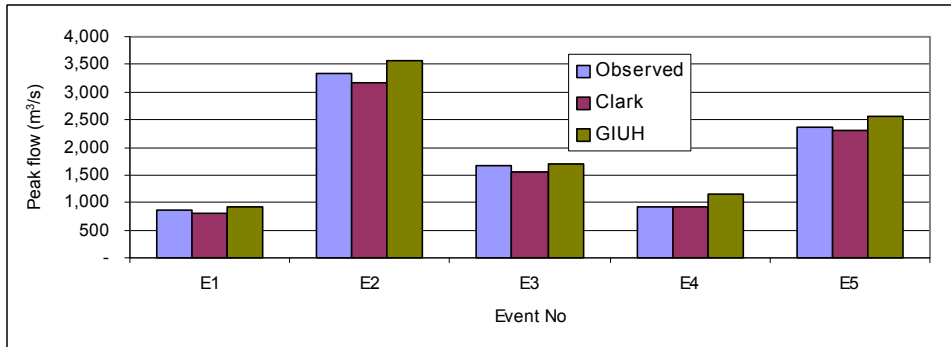


Fig. 9: Comparison between observed and computed values of peak flows (calibration).

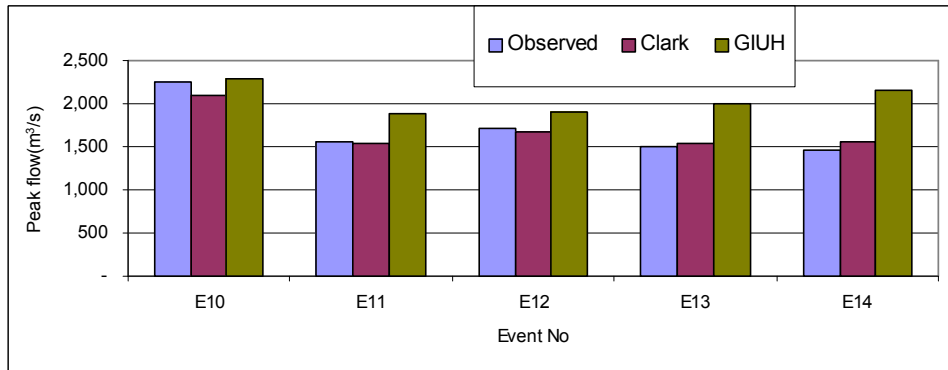


Fig. 10: Comparison between observed and computed values of peak flows (validation).

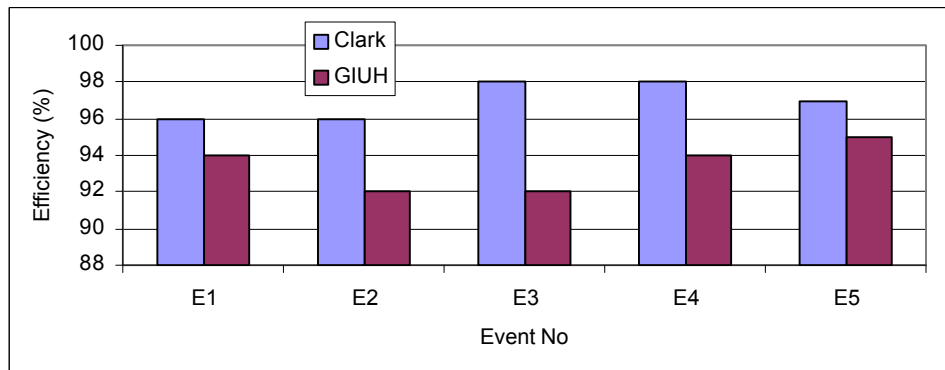


Fig. 11: The model efficiency for sample events.

Table 1: Model Errors and Performance

Event		% Error				Model Performance			
		% $\Delta V$		% $\Delta q_p$		% $\Delta t_p$		% $\eta$	
		Clark	GIUH	Clark	GIUH	Clark	GIUH	Clark	GIUH
1	Calibration	-2	13	-5	9	0.25	2	96	94
2		3	6	-6	7	-1	3	96	92
3		4	10	-6	2	0.15	3	98	92
4		12	10	-15	8	-3	8	98	94
5		2	2	-10	6	0.3	10	97	95
6		11	12	-19	10	0	8	97	93
7		4	1	-11	16	-3	13	98	92
8		-2	14	-3	15	0	4	98	95
9		7	15	-13	13	0	5	98	89
10		-3	12	-4	10	-2	6	98	93
11	Validation	5	12	-10	13	0	0	98	92
12		5	1	-11	2	-2	8	98	95
13		8	9	-14	17	0	14	97	88
14		13	18	-19	19	-1	7	97	87
15		8	5	-12	5	0	8	98	95

the error in time to peak ( $\Delta T_p$ ) and runoff volume (%  $\Delta V$ ). Over all error in simulated results are given in Table 1. The performance of two models is given in Figure 11. Table 1 shows that the Clark model simulated better results as compared to those of the GIUH model for most of the features. The model efficiency (coefficient of determination  $\eta$ ) is higher for Clark model as compared to the GIUH.

As shown by Table 1, the time required to attain peak flow is comparatively better simulated by the Clark model as compared to the GIUH. Actually the time area diagram, which is based on time of concentration, takes care of time-based movement of surface runoff at sub catchment level. In case of GIUH, the Horton order ratios describe catchment topology at sub catchment level, but the errors still exist due to the fact that in the Strahler's ordering scheme as shown in Figure 2, the drainage arrangements do not capture true picture in large catchments. Better results may be obtained in such cases if the drainage pattern of catchment is defined in more detailed manner. The number of geomorphologic descriptors needs also to be increased for better physical realization of the watershed. One thing very important to be noted is that the scale of a satellite image has strong impact on the conceived drainage pattern. In the GIUH model the impact of dead areas that do not contribute to surface runoff is also not taken into account.

The surface runoff volume is also comparatively better simulated by Clark model as compared to GIUH. The better precision of Clark model in this regard is due to

the fact that it better estimates the recession part of DSRH as compared to GIUH. The error in simulating peak flow is nearly same in both the models.

## CONCLUSIONS

The Clark model simulate better results than those of the GIUH model for simulation of direct runoff hydrograph for most of its features. Time to peak and the volume of direct runoff is better simulated by the Clark model as compared to the GIUH. The error in peak discharge is similar in the results simulated by both the models. Overall performance of the Clark model represented by the coefficient of determination is higher than that of the model based on GIUH. Coefficient of determination on the average is 98% in case of the Clark model. Arc GIS and ERDAS are the useful tools for digitization of the catchment maps for limited stream order using SPOT satellite images. However, it is highly laborious and time taking task to get the geographical parameters for large catchments with higher stream order.

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